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Swift pointing and gravitational-wave bursts from gamma-ray burst events

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Abstract

The currently accepted model for gamma-ray burst phenomena involves the violent formation of a rapidly rotating solar-mass black hole. Gravitational waves should be associated with the black-hole formation, and their detection would permit this model to be tested. Even upper limits on the gravitational-wave strength associated with gamma-ray bursts could constrain the gamma-ray burst model. This requires joint observations of gamma-ray burst events with gravitational and gamma-ray detectors. Here we examine how the quality of an upper limit on the gravitational-wave strength associated with gamma-ray bursts depends on the relative orientation of the gamma-ray-burst and gravitational-wave detectors, and apply our results to the particular case of the Swift Burst-Alert Telescope (BAT) and the LIGO gravitational-wave detectors. A result of this investigation is a science-based 'figure of merit' that can be used, together with other mission constraints, to optimize the pointing of the Swift telescope for the detection of gravitational waves associated with gamma-ray bursts.

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1. Introduction

The currently accepted model for gamma-ray burst phenomena involves the violent formation of a rapidly rotating approximately solar-mass black hole surrounded by a similarly massive debris torus [1, 2]. A gravitational-wave burst is likely to be associated with the formation of this 'central engine', and the observation of such a gravitational-wave burst may reveal details of the central engine that cannot be revealed through observations of the gamma rays alone. In this paper we examine how joint observations from the LIGO gravitational-wave detectors [3] and the Swift gamma-ray burst satellite [4] can be used to detect or place upper limits on gravitational-wave emission by gamma-ray burst events.

Finn *et al* [5] have described how the cross-correlated output of two gravitational-wave detectors, taken in coincidence with gamma-ray burst (GRB) events, can be used to detect or place upper limits on the emission of gravitational-wave bursts (GWBs) by GRBs. Finn *et al* estimate that 1000 GRB observations combined with observations from the initial LIGO detectors could produce an upper limit on the gravitational-wave strain associated with GRBs of approximately $h_{\mathrm{RMS}} \leqslant 1.7 \times 10^{-22}$ at 95% confidence.

In their original work Finn *et al* assumed that GRBs would be detected isotropically; i.e., that the GRB detector had an isotropic antenna pattern. They did note, however, that the Swift satellite [4], a next-generation multi-wavelength satellite dedicated to the study of GRBs, does not have an isotropic antenna pattern and that this has potentially important consequences for the ability of the combined GRB/GWB detector array to detect or limit the gravitational-wave flux on Earth owing to GRBs. Here we study this question specifically in the context of the Swift satellite and the LIGO gravitational-wave detectors; i.e., we determine, as a function of Swift's pointing, the sensitivity of the Swift/LIGO detector array to gravitational waves from GRBs, and propose a figure of merit that can be used in Swift mission scheduling to optimize the sensitivity of the Swift/LIGO array to the gravitational-wave flux from GRBs. We find that the upper limit that can be placed on h_{RMS} differs by a factor of 2 between best and worst orientations of the satellite.

In section 2 we review the analysis of Finn *et al* [5], focusing on the statistic that is used to detect or bound the gravitational-wave strength associated with gamma-ray bursts. In section 3 we extract the direction dependence of this statistic and evaluate it specifically for the case of the LIGO detectors and the Swift Burst Alert Telescope. We conclude with some brief remarks in section 4.

2. Observing a GRB–GWB association

In this section we briefly review the analysis presented in [5].

Consider a set of N GRB detections, each characterized by the direction to the source $\hat{\Omega}_k$ and the arrival time τ_k of the burst at the Earth's barycentre. A plane gravitational wave incident on the detector pair from the direction $\hat{\Omega}_k$ will lead to correlated detector responses with a time lag equal to

$$\Delta t_k = t_k^{(2)} - t_k^{(1)}, \tag{1}$$

where $t_k^{(i)}$ is the arrival time of the burst at detector i, which depends only on τ_k, $\hat{\Omega}_k$ and the detector location.

Let $s_i(t)$ be the calibrated output of gravitational-wave detector $\mathcal{D}_i$, which we assume to consist of detector noise $n_i(t)$ and possibly a gravitational-wave signal $h_i(t)$ produced by the GRB source:

$$s_i(t) = n_i(t) + h_i(t). \tag{2}$$

Finn *et al* [5] define

$$S(\hat{\Omega}_k, \tau_k) = \langle s_1, s_2 \rangle := \int_0^T \mathrm{d}t \int_0^T \mathrm{d}t' s_1\big(t_k^{(1)} - t\big) Q(t - t') s_2\big(t_k^{(2)} - t'\big), \tag{3}$$

as a measure of the cross-correlation of the two detectors corresponding to the GRB characterized by $(\hat{\Omega}_k, \tau_k)$. Here Q is a freely specifiable symmetric filter function, and T

is chosen large enough to encompass the range of possible times by which the gravitational waves from a GRB event may precede the gamma rays, which is typically thought to be of order 1 s for GRBs produced by internal shocks and 100 s for GRBs produced by external shocks [6, 7].

The presence of a GWB in the data will increase the mean value of the cross-correlation (3) over its mean value when no GWBs are present. Finn *et al* [5] showed that this effect can be used to detect or place upper limits on the gravitational-wave strain associated with GRBs by comparing the mean value S_{on} of the cross-correlation statistic for a set of GRB observations to the mean cross-correlation S_{off} computed using random $(\hat{\Omega}, \tau)$ not associated with any GRB. The expected difference between S_{on} and S_{off} is simply the contribution due to GWBs from the GRB events; using (2), we have

$$S_{\text{on}} - S_{\text{off}} = \overline{\langle h_1, h_2\rangle} = \int_0^T \mathrm{d}t \int_0^T \mathrm{d}t'\, Q(t-t')\overline{h_1\big(t_k^{(1)} - t\big)h_2\big(t_k^{(2)} - t'\big)}, \tag{4}$$

where the overline denotes an average over the population of GRBs. Since the right-hand side of (4) is quadratic in the gravitational-wave strain, a measurement of or limit on the difference in cross-correlations leads directly to an estimate of or upper limit on the gravitational-wave strain associated with GRBs.

Here we are not interested in the absolute value of the upper limit that can be achieved, but rather in how that upper limit varies according to the relative orientation of the GWB/GRB detector array. We shall extract this dependence in the next section.

3. Direction dependence of the upper limit

The analysis described in [5] involved two gravitational-wave detectors. It assumed for simplicity that these two detectors had identical isotropic antenna patterns and that each was sensitive to exactly the same gravitational-wave polarization. Here we relax all of these approximations; i.e., we properly account for the position and orientation of the gravitational-wave detectors, focusing particularly on the two LIGO detectors on the Earth, and the dependence of their sensitivity to the direction to the GRB source. Our result is an expression for the dependence of the upper limit on the population-averaged gravitational-wave strength $\overline{\langle h_1, h_2\rangle}$ as a function of the distribution of *detected* GRBs on the sky. We also combine this result with the directional sensitivity of the Swift detector to determine the dependence on Swift pointing of the upper limit on $\overline{\langle h_1, h_2\rangle}$ that can be set by joint LIGO/Swift observations.

The gravitational-wave contribution h_i to the output of the ith LIGO detector is a linear function of the physical gravitational-wave strain $h_{ab}(t, \vec{x})$,

$$h_i(t) = h_{ab}(t, \vec{x}_i)\, d_i^{ab}, \tag{5}$$

where $\vec{x}_i$ is the gravitational-wave detector's location and d_i^{ab} is the detector response function. For interferometer i with arms pointing in the directions given by unit vectors $\hat{X}_i$, $\hat{Y}_i$, the latter is

$$d_i^{ab} = \tfrac{1}{2}\big(X_i^a X_i^b - Y_i^a Y_i^b\big). \tag{6}$$

It is convenient to decompose the gravitational wave into its two polarization states,

$$h_{ab}(t) = h_+(t)\epsilon^+_{ab}(\hat{\Omega}) + h_\times(t)\epsilon^\times_{ab}(\hat{\Omega}). \tag{7}$$

See [8] for one choice of the polarization tensors $\epsilon^A_{ab}(\hat{\Omega})$. Lacking any detailed model for the gravitational waves that may be produced in a GRB event, we assume that (i) the waves have equal power in the two polarizations ($\overline{h_+(t)h_+(t')} = \overline{h_\times(t)h_\times(t')}$) and (ii) the two polarizations are uncorrelated ($\overline{h_+(t)h_\times(t')} = 0$).

We focus attention now on the mean gravitational-wave contribution $\overline{\langle h_1, h_2\rangle}$ to the cross-correlation statistic (3). For GWBs arriving from the direction $\hat{\Omega}$, it can be shown that

$$\overline{\langle h_1, h_2\rangle} = \rho_{\mathrm{GWB}}(\hat{\Omega}|d_1, d_2) \int_0^T \mathrm{d}t \int_0^T \mathrm{d}t' Q(t-t') \overline{h_+(t) h_+(t')}, \tag{8}$$

where

$$\rho_{\mathrm{GWB}}(\hat{\Omega}|d_1, d_2) \equiv \sum_{A=+,\times} d_1^{ab} \epsilon_{ab}^A(\hat{\Omega}) d_2^{cd} \epsilon_{cd}^A(\hat{\Omega}) \tag{9}$$

describes the direction dependence of the sensitivity of the gravitational-wave detector pair to the GWB.

To complete the evaluation of $\overline{\langle h_1, h_2\rangle}$ consider the fraction of GRB detections that arise from different patches on the sky. Since the intrinsic GRB population is isotropic, the distribution of detection on the sky depends entirely on the directional sensitivity of the GRB detector. Let the fraction of GRB detections in a sky patch of area $d^2\hat{\Omega}$ be given by

$$\rho_{\mathrm{GRB}}(\hat{\Omega}|\hat{\Omega}', \hat{n}')\, d^2\hat{\Omega}, \tag{10}$$

where $\hat{\Omega}'$ is the direction in which the GRB detector is pointed, and $\hat{n}'$ describes the rotation of the satellite about its pointing direction[6]. It can then be shown that the upper limit on the squared gravitational-wave strain averaged over the observed GRB population when the orientation of GWB and the GRB detectors are given by $(\hat{\Omega}', \hat{n}', d_1, d_2)$ is inversely proportional to

$$\zeta(\hat{\Omega}', \hat{n}', d_1, d_2) = \int d^2\hat{\Omega} \rho_{\mathrm{GRB}}(\hat{\Omega}|\hat{\Omega}', \hat{n}') \rho_{\mathrm{GWB}}(\hat{\Omega}|d_1, d_2). \tag{11}$$

Clearly ζ can be regarded as a figure of merit that describes how capable the gravitational-wave/gamma-ray burst detector combination is at identifying GWBs associated with GRBs as a function of the detector orientations. This figure of merit may be normalized to have a maximum of unity; however, regardless of the normalization

$$[\zeta(\hat{\Omega}', \hat{n}', d_1, d_2)/\zeta(\hat{\Omega}'', \hat{n}'', d_1, d_2)]^{-1} \tag{12}$$

is the ratio of the upper limits on the squared gravitational-wave amplitude that can be attained by orienting the GRB satellite as $(\hat{\Omega}', \hat{n}')$ versus $(\hat{\Omega}'', \hat{n}'')$. To the extent that, e.g., the GRB detector orientation $(\hat{\Omega}', \hat{n}')$ can be manipulated on orbit, choosing orientations that maximize ζ will lead to larger signal contributions (4) to the cross-correlation and thus more sensitive measurements of the gravitational-wave strength associated with GRBs.

Let us now consider the special case of the Burst Alert Telescope (BAT) on the Swift satellite [4] and the LIGO gravitational-wave detectors [3]. The BAT is a wide field-of-view coded-aperture gamma-ray imager that will detect and locate GRBs with arc-minute positional accuracy. Its sensitivity to GRBs depends on the angle λ between the line of sight to the GRB and the BAT axis, as well as the rotational orientation of the satellite about the BAT axis. The BAT sensitivity averaged over the azimuthal angle as a function of λ has been evaluated by the BAT instrument team. It has the approximate form[7]

$$\rho_{\mathrm{Swift}} = \begin{cases} 2\cos\lambda - 1 + 0.077\sin[13(1-\cos\lambda)] & \lambda \in [0, \pi/3], \\ 0 & \text{otherwise}. \end{cases} \tag{13}$$

For the purposes of illustration we will use this azimuthal-angle averaged expression for the BAT sensitivity.

[6] The detailed form of ρ_{GRB} depends on the detailed construction of the GRB detector. We consider the case of the Swift Burst Alert Telescope here.

[7] See the BAT section of the Swift homepage, http://swift.gsfc.nasa.gov/science/instruments/bat.html.

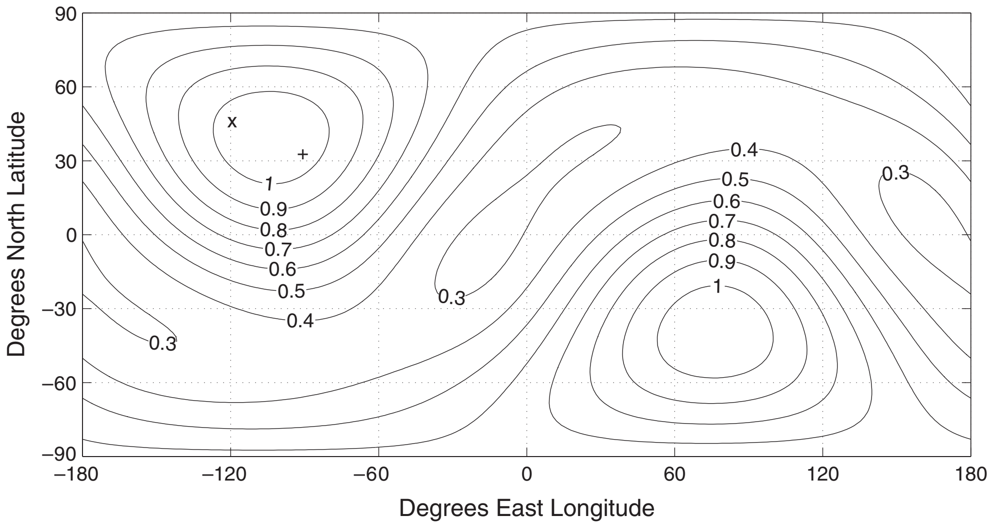

Figure 1. Figure of merit ζ (11) for Swift pointing in Earth-based coordinates, produced by convolving the LIGO sensitivity pattern ρ_{GWB} (9) with the Swift sensitivity function ρ_{Swift} (13). The figure of merit is nowhere zero, having a range of [0.25, 1.00] and an all-sky average of 0.56. The + and × mark the locations of the LIGO Livingston and LIGO Hanford detectors.

Convolving ρ_{GWB} (9) for the LIGO detector array (see [9]) with the Swift sensitivity function ρ_{Swift} as in equation (11) gives the figure of merit ζ for the Swift pointing, which is shown in figure 1. The figure of merit is nowhere zero, varying by a factor of approximately 4 between best (near zenith of detectors) and worst (near planes of detectors at 45° from arms) orientations of Swift; this translates into a factor 2 difference in amplitude sensitivity. The all-sky average of the figure of merit is 0.56 times the maximum value.

4. Conclusion

In this paper, we have evaluated how the quality of an upper limit on the gravitational-wave strength associated with gamma-ray burst observations depends on the relative orientation of the gamma-ray burst and gravitational-wave detectors, with particular application to the Swift Burst-Alert Telescope (BAT) and the LIGO gravitational-wave detectors. Setting aside other physical and science constraints on the Swift mission, careful choice of BAT pointing leads to an upper limit on the observed GRB population-averaged mean-square gravitational-wave strength a factor of two lower than the upper limit resulting from pointing that does not take this science into account.

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