GRAVITATION

Time travel on a string

Bruce Allen and Jonathan Simon

TIME machines are the stuff of good science fiction and bad Hollywood movies — one would think. But about a year ago, J. R. Gott proposed[1] a simple way to build such a machine out of two long straight pieces of the exotic form of matter called cosmic strings. Whether cosmic strings really exist is not known, but that does not diminish the elegance of Gott's design, which does not seem to violate any known laws of physics. The enthusiastic debate over whether such a machine might indeed be built and function as advertised has stimulated several highly respected research groups to examine the relevant fundamental principles in much greater detail[2–5]. Indeed, Hawking has formulated a "Chronology Protection Conjecture"[5] which, if correct, would rule out time machines altogether. In fact, the parts that have already been proved suggest that attempts to build a time machine from Gott's idea are doomed.

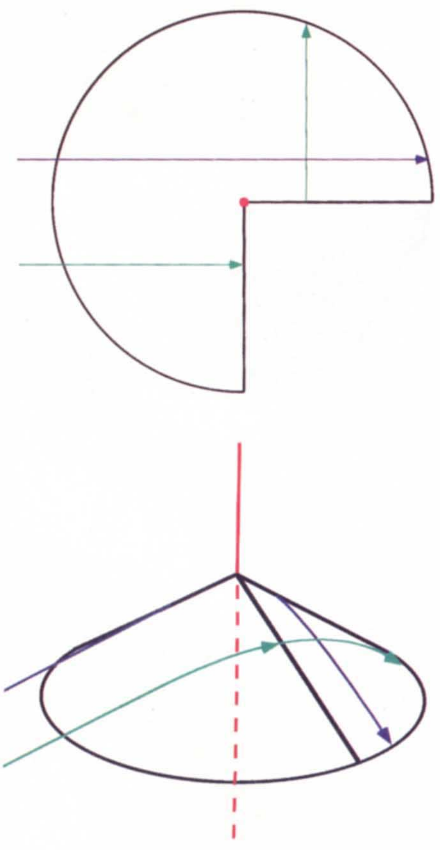

FIG. 1 The gravitational field of a cosmic string (red) gives space a conical geometry. At a fixed time, if the string runs vertically through the page, the cone may be visualized as a circular disk with a wedge removed (top). The sides of the wedge are then glued together (bottom). The green path is a continuous one that vaults the wedge. In the remaining diagrams, only the edges of the excised wedges are shown. Note that although the blue and green paths start parallel, they eventually converge and intersect.

The notion of a time machine is simple enough. Fortifying yourself with a freshly brewed cup of coffee, you step into it at 9 AM. A few seconds later (according to your wristwatch) you step out again, finding yourself surrounded by the aroma of coffee brewing, with the clock on the wall reading 8.30 AM, that same morning. Seating yourself comfortably in the corner, you can now watch yourself drink your coffee and step into the time machine at precisely 9 AM.

The enigmatic nature of this situation is familiar to even the most casual reader of science fiction. Suppose the time traveller knocks over the coffeepot at 8.45 AM, or disables the time machine, or even murders the other copy of himself? Then the time traveller could prevent himself from making the voyage: a clear contradiction or paradox. For this reason, it has generally been thought that time travel is impossible. But work by Friedman, Thorne and others[6–8] has established the likelihood that a "consistent" solution is possible in general — in this case, one for which the time traveller, stepping into his machine, sees another copy of himself sitting quietly in the corner chair, watching. As long as there is at least one consistent solution, the question of free will may be quietly ducked, just as it is for the ordinary, deterministic physics of Newton.

The question "can one build a time machine?" can be posed precisely in the language of Einstein's theory of general relativity. This theory describes space (x, y, z) and time (t) as a unified four-dimensional continuum (t, x, y, z) distorted and bent by the gravitational field of matter. Any object follows a curve through this spacetime; for example a person sitting in a chair at the space point $x=y=z=0$ follows a 'timelike' curve described by the line consisting of the points (all t, $x=y=z=0$). Viewed this way, the time-traveller follows a curve through spacetime which starts at some initial spacetime point (t, x, y, z), moves forward in time (as measured by the traveller's watch) but which nevertheless returns to the same spacetime point (t, x, y, z) along a 'closed timelike curve'. Because it is matter that determines the way that spacetime curves, the crucial question is: can one generate closed timelike curves using only the different types of matter that exist?

Four years ago, Morris and Thorne[9,10] designed a time machine based on the Casimir effect. The Casimir effect produces a region in which the energy of all the matter contained is negative (and so also the mass, according to Einstein's famous equation $E = mc^2$). Roughly speaking, this is possible because even empty space has free energy due to quantum fluctuations. With electrically conducting boundaries (metal plates), it is possible to suppress some of these fluctuations, reducing the energy below the zero-point level. Time machines of this type, which rely on negative energy sources, are not as troubling as Gott's time machine, because known physical laws might well conspire to keep negative energy from building up to the degree necessary to allow closed timelike curves.

Gott's design, which relies on cosmic strings, does not suffer this way. In 1976, Kibble discovered[11] that such matter is predicted by 'gauge' theories, the best existing mathematical description of the behaviour of matter at very high energies and densities. Astronomers have so far failed to find any cosmic strings, but nevertheless they may have been significant in allowing large-scale cosmological structures like galaxies to develop.

Although their diameter would be only 10^{-29} cm, cosmic strings would typically pack 10^{22} g into each centimetre of their length. In the early 1980s, Gott[12] and Hiscock[13] independently discovered that cosmic strings would bend and distort spacetime in a very peculiar way. The spacetime outside the string is uncurved, or flat (there are no gravitational forces), as if no string were present. But the circumference of a circle centred on the string is less than the euclidean circumference, $2\pi r$, as if a wedge of angle α had been cut out of a circular sheet of paper, and the edges glued together to form a cone (Fig. 1). The angle α, the deficit angle, would be about 1/10,000 of a degree for a realistic cosmic string (in our figures we take α to be 90° to exaggerate the effects). While the flat space outside the string is locally no different from what it would be in the absence of the string, its global properties are affected.

For example, two initially parallel light rays that pass by the string on opposite sides become focused together and eventually cross paths (Fig. 1, bottom). Gott's time machine, constructed with two infinitely long, straight cosmic strings, moving at high velocity (Fig. 2), exploits this ability of strings to bend spacetime and to focus light rays. To see how it works, one need only understand first how objects (or observers) behave in a spacetime with a cosmic string, and second how Einstein's theory of special relativity changes the way time is measured by observers moving at different speeds.

As described above, the space around

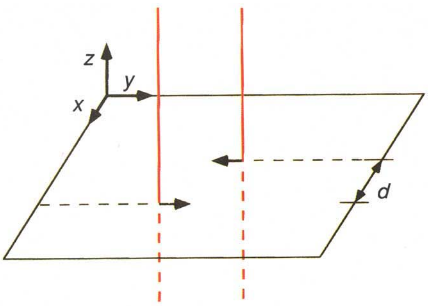

FIG. 2 A snapshot of Gott's two infinite strings (red) just before their closest approach (*d*, closest separation). In Fig. 3, the *z* direction is not shown.

a cosmic string is like a cone whose vertex lies on the string itself: a circle drawn around the cone has a circumference less than 2π times the distance to the vertex. Equivalently, one may imagine moving on a flat piece of paper with a wedge cut out, with the additional rule that any object travelling along the surface of the paper, when it hits the border of the wedge, instantaneously transports across the wedge to the other side, leaving at the same relative angle as it entered (Fig. 1).

This two-dimensional example is closely related to the four-dimensional cosmic-string spacetime. If one takes a cross-section of the spacetime through the string at a given instant of time, the cross-section of the string itself corresponds to the vertex of the cone (or of the missing wedge). An infinite stack of cones would correspond to an infinitely long cosmic string, where the vertices of the cones run along the string. The same would apply to an infinite stack of paper with a wedge removed from every sheet, and the vertices running along the string. Letting time evolve naturally brings back the fourth dimension of time.

The fundamental postulate of Einstein's special relativity is that the speed of light in vacuum is the same for all observers (unlike the speed of sound, which changes for an observer moving with respect to the background air). This leads to a loss of simultaneity: two events occurring at the same time as measured by one observer at rest occur at different times for a moving observer.

This has an important effect for moving cosmic strings. One can build a stationary string by stacking up sheets of paper with the wedges removed, plus the instantaneous-transport rule. But if the string is moving, the instantaneous-transport rule must change according to special relativity: points on opposite sides of the wedge on a single sheet are simultaneous only for an observer for whom the string is stationary. For an observer who sees the string speed by, the identified points on opposite sides of the wedge lie at different times. If the string is moving to the left, then transporting across the wedge to the right results in a jump backwards in time (and transporting across the wedge to the left results in a jump forwards; see Fig. 3).

As mentioned before, Gott's time machine is based on two cosmic strings moving in opposite directions. If there were only one string, there would never be enough time to return to one's initial position and initial time (without going faster than the speed of light, which would violate the laws of special relativity). This can be seen most clearly from the point of view of an observer moving at the same velocity as the string, for whom the string is stationary and for whom there is no time jump at all. If there are no closed timelike curves for this observer, then there are no closed timelike curves for any observer.

With two cosmic strings moving in different directions the situation is altered, because no observer can see both strings at rest at once. A closed timelike curve can easily be traversed by a time traveller who first moves around one string in the direction opposite to that string's direction of travel, then turns around and moves around the other string, opposite to its direction of travel, ending up at the starting point (the turnarounds appear in Fig. 3 as corners on the green curves; these accelerations are necessary because there are no closed timelike geodesics). If the strings are moving quickly enough, then because both time jumps are backwards, the observer can return to the same initial position and to the same initial time. The strings' speed necessary for this to work is related to the angle of the wedge, as the larger the wedge, the larger a time jump is accomplished. The actual relation between the two is that the speed v, as measured by an observer who sees the strings coming together with equal but opposite velocities, must be larger than $c\cos(\alpha/2)$ where c is the

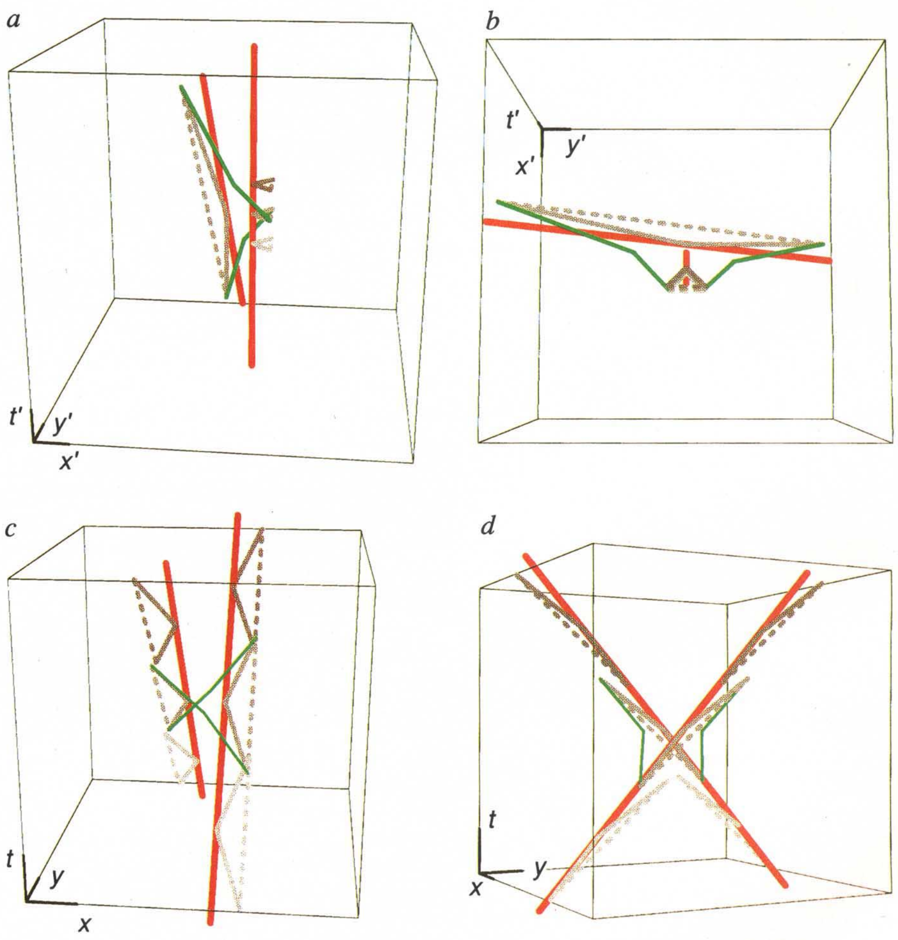

FIG. 3 Gott's time machine as seen by an observer at rest next to one of the strings (*a*, *b*) and by an observer who sees the two strings rushing together, with equal but opposite velocities, 0.89*c* (*c*, *d*). The time coordinate in each case runs directly upwards. The cosmic strings are red, the excised wedges are different shades of grey, and the closed timelike curve is green. Note that in *a* and *b*, the 90° excised wedges of the stationary string (right) are horizontal (t' = constant) surfaces. The wedges associated with the other string, which is moving, are stretched and its glued edges connect points at different times, because of the loss of simultaneity in special relativity. Owing to the effects of special relativity, the wedges in parts *c* and *d* are not surfaces of constant time (horizontal) for either string. Parts *b* and *d* are the same as parts *a* and *c*, respectively, but are seen from a different perspective.

speed of light. In our diagrams with $\alpha = 90°$, v must be larger than $0.71c$ to make a time machine. In Fig. 3, the strings are moving with $v = 0.89c$.

The timing of a trajectory such as the one just described is critical: if the observer starts at the wrong place and time, time travel is impossible. Starting out near the point of closest approach of the strings, but either too early or too late, one cannot complete the trip at the initial time and place of departure without going faster than light. One may start far away from the point of closest approach, but then one must be going close to the speed of light to get there in time to circle both strings. Cutler[4] has found explicitly which starting points allow travel along closed timelike curves and which do not. The regions where closed timelike curves exist form a collar around the location and time of closest approach (the central region of Fig. 3*c*, *d*). The boundary of the regions comes in and exits at the speed of light. In this sense, there are no closed timelike curves in the far past and far future of the spacetime. Furthermore, one could create closed timelike curves using strings which have travelled for only a limited period of time.

Headway

As cosmic strings are considered a realistic form of matter, and time travel would be quite an achievement, considerable effort has gone into looking for obstacles that might prevent one constructing or operating Gott's design. Headway has been made because a system of parallel strings moving in four spacetime dimensions is mathematically indistinguishable from a system of point masses interacting gravitationally in three spacetime dimensions. Exploiting this analogy (first pointed out by Gott[12]), Deser, Jackiw and 't Hooft show[3] that, although the masses corresponding to each cosmic string move at less than the speed of light, what they call the total momentum (in three space-time dimensions) would correspond to a single string moving faster than light. Such systems are called tachyonic, from the Greek for fast.

It is not clear what the repercussions of this may be, but it does appear to be closely tied to the possibility of time travel. If the strings slow down enough to eliminate the closed timelike curves, the momentum of the system ceases to be tachyonic. One might think that it would be enough to create a tachyonic subsystem in isolation, but this appears insufficient, because if any closed timelike curves exist, it may be shown that they are not confined to any finite region (in agreement with Cutler). Thus this work suggests that although the cosmic strings themselves are a reasonable form of matter, the combined system of two such infinite strings, moving at high relative velocity, might never be obtained in our Universe. These results are only suggestive, however, as the exact relationship between momentum in three spacetime dimensions and in four spacetime dimensions is still not clear. For example, in three spacetime dimensions the momentum of the system of strings is tachyonic, but the motion of the centre of mass of the system is non-tachyonic. In four spacetime dimensions they should be the same.

Using somewhat different methods, Carroll, Farhi and Guth[2] reach the same conclusion. They analyse a system containing several cosmic strings, again exploiting the parallel with a system of masses in three spacetime dimensions. The geometry of such a system is characterized by a universal quantity (known as the holonomy) which describes the way in which a vector changes direction after being carried around a closed curve. The change in direction is specified by a quantity related to the momentum of the combined system of strings, which Carroll, Farhi and Guth show corresponds to tachyonic matter (when closed timelike curves are present). They also examine several likely ways of creating systems with tachyonic momentum, but they cannot do so starting with non-tachyonic matter. Again, this suggests that Gott's time machine may be fundamentally out of reach.

There are even more obstacles. In constructing a mathematical description of physical systems, one generally makes idealizations of some kind. In a laboratory, cosmic strings of infinite length would not be available, but very long portions of strings might be used (or even two halves of a very long and almost-straight loop of cosmic string). Would Gott's time machine still work if the infinitely long cosmic strings were replaced with very long, but finite ones? Gott suggests that any closed timelike curves created with finite lengths of string would form a black hole and then fall into it before they could be exploited. The work by Carroll, Deser and their collaborators does not directly address this question, although they show that there is something suspect about the global geometry (related to the tachyonic nature of the system). Effectively, Gott's machine requires spacetime to have unusual large-scale geometry, and unfortunately for the aspiring inventor, nothing one does in a laboratory can alter this large-scale geometry.

Conjecture

In a bold attempt to rule out time machines altogether, Hawking now puts forward what he calls "The Chronology Protection Conjecture"[5]. To support this grand conjecture, he has proved a theorem which shows that if one constructs a time machine out of very long but not infinite strings, then either one must surround the time machine with a shell of negative energy, or singularities of the same sort that one finds at the centre of black holes would form, shutting off access to the time machine (and destroying the laboratory in the process). This seems to offer a definitive answer to the question of whether one could build a time machine in a laboratory based on Gott's design, and whether it would work.

Unfortunately the answer is no (without resorting to the use of negative energy). Although Gott's design appears to be based on a realistic type of matter (cosmic strings), actually to build one that would not disappear into a black hole would apparently require large amounts of (unrealistic) matter with negative energy. Even so, if our Universe is infinitely large and infinite cosmic strings naturally formed after the Big Bang (as predicted by several cosmological models), then the possibility that the Universe itself created a time machine cannot be ruled out.

While there is still hope that one day a sufficiently clever design may make building a time machine possible, it is beginning to seem more and more improbable. Like the perpetual motion machines of the nineteenth century, the designs have an elegant simplicity (as well as enormous commercial potential), but it seems that Nature may abhor them just as much. □

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Correction

THE scanning tunnelling micrograph of fullerene molecules by Y. Zhang *et al.*, which appeared in News and Views recently (*Nature* **356**, 383) first appeared in the *Journal of Physical Chemistry* (**96**, 510–513; 1992), not the *Journal of the American Chemical Society*, as reported.